Strassen's Matrix Multiplication

Follow Divide and conquer to multiply two square matrix using some formula.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

A B C

A, B, C are square matrix of size $N \times N$

a, b, c, d are submatrix of matrix A of size $\frac{N}{2} \times \frac{N}{2}$

e, f, g, h are sub matrix of B of size $\frac{N}{2} \times \frac{N}{2}$

complexity $\Rightarrow O(N^3)$

C 8 multiplication of $\frac{N}{2} \times \frac{N}{2}$ and 4 addition

$$T(N) = 8\,T\left(\frac{N}{2}\right) + O(N^2)$$

complexity

$$T(N) = 7T\left(\frac{N}{2}\right) + O(N^2)$$

$$= O(N^{2.8074})$$

Stressen's Method reduce the no. of recursive call to 7 by using following formula

$P_1 = a(f-h)$

$P_2 = (a+b)h$

$P_3 = (c+d)e$

$P_4 = d(g-e)$

$P_5 = (a+d)(e+h)$

$P_6 = (b-d)(g+h)$

$P_7 = (a-c)(e+f)$

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} = \begin{bmatrix} \cancel{P_5+P_4+P_2}+P_6 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{C}$$

$$\begin{bmatrix} P_5+P_4-P_2+P_6 & P_1+P_2 \\ P_3+P_4 & P_1+P_5-P_3-P_7 \end{bmatrix}$$

A, B, C are square matrix of size $N \times N$

a, b, c, d are submatrices of A, of size $\frac{N}{2} \times \frac{N}{2}$

e, f, g, h are submatrices of B, of size $\frac{N}{2} \times \frac{N}{2}$

$P_1, P_2, P_3, P_4, P_5, P_6$ and P_7 are submatrices of size $\frac{N}{2} \times \frac{N}{2}$